Continuous Fourier Transform Correspondence Table

$$\mathcal{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t)\,\mathrm{e}^{-\mathrm{j}\omega t}\,dt, \qquad f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,\mathrm{e}^{\mathrm{j}\omega t}\,d\omega$$

Notes: We use $\mathrm{j} = \sqrt{-1}$. Distributions appear for non-absolutely integrable signals (e.g., δ and $\epsilon(t)$); p. v. denotes Cauchy principal value. We adopt $\mathrm{sinc}(x) = \dfrac{\sin x}{x}$ (non-π-normalized).

A. Common Transform Pairs (this convention)

Time domain $f(t)$	**Frequency domain** $F(\omega)$	**Notes / Conditions**
$\delta(t)$	1	Unit impulse
1	$2\pi\,\delta(\omega)$	Constant signal
$\mathrm{e}^{\mathrm{j}\omega_0 t}$	$2\pi\,\delta(\omega-\omega_0)$	Spectral line at ω_0
$\cos(\omega_0 t)$	$\pi\big[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\big]$	Real even line pair
$\sin(\omega_0 t)$	$\frac{\pi}{\mathrm{j}}\big[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\big]$	Imag. odd line pair
$\epsilon(t)$	$\pi\,\delta(\omega) + \mathrm{p.v.}\big(\frac{1}{\mathrm{j}\omega}\big)$	Step function (distribution)
$\mathrm{sgn}(t)$	$\frac{2}{\mathrm{j}\omega}$ (as p. v.)	$\mathrm{sgn}(t) = 2\epsilon(t) - 1$
$\mathrm{e}^{-a\|t\|},\ a>0$	$\frac{2a}{a^2+\omega^2}$	Two-sided decaying exponential
$\mathrm{e}^{-\frac{t^2}{2\sigma^2}}$	$\sqrt{2\pi}\,\sigma\,\mathrm{e}^{-\frac{\sigma^2\omega^2}{2}}$	Gaussian (self-FT up to scale)
$\mathrm{rect}\big(\frac{t}{T}\big)$	$T\,\mathrm{sinc}\big(\frac{\omega T}{2}\big)$	$\mathrm{rect}(x) = 1$ for $\|x\| < \frac{1}{2}$
$\mathrm{tri}\big(\frac{t}{T}\big)$	$T\,\mathrm{sinc}\big(\frac{\omega T}{2}\big)^2$	Triangular pulse
$\frac{\sin(\Omega t)}{t}$	$\pi\,\mathbf{1}_{[-\Omega,\Omega]}(\omega)$	Ideal low-pass (rect band), $\Omega > 0$
$\frac{1}{t}$	$-\mathrm{j}\pi\ \mathrm{sgn}(\omega)$ (as p. v.)	Odd distribution
$(\epsilon(t+\frac{T}{2}) - \epsilon(t-\frac{T}{2}))$	$2\,\frac{\sin(\omega T/2)}{\omega}$	Rect pulse, width T

B. Useful Properties

Time domain	**Frequency domain**	**Comments**
Linearity: $\mathcal{F}\{af+bg\}$	$aF+bG$	$a, b \in \mathbb{C}$
Time shift: $f(t-t_0)$	$\mathrm{e}^{-\mathrm{j}\omega t_0}F(\omega)$	Delay $t_0 \in \mathbb{R}$
Frequency shift (modulation): $\mathrm{e}^{\mathrm{j}\omega_0 t}f(t)$	$F(\omega-\omega_0)$	Shift by ω_0
Scaling: $f(at)$	$\frac{1}{\|a\|}F\big(\frac{\omega}{a}\big)$	$a \in \mathbb{R}\setminus\{0\}$
Convolution: $(f*g)(t)$	$F(\omega)G(\omega)$	$(f*g)(t) = \int f(\tau)g(t-\tau)\,d\tau$
Multiplication: $f(t)g(t)$	$\frac{1}{2\pi}(F*G)(\omega)$	Frequency convolution
Differentiation: $\frac{d^n}{dt^n}f(t)$	$(\mathrm{j}\omega)^n F(\omega)$	$n \in \mathbb{N}$
Time moment: $t\,f(t)$	$\mathrm{j}\,\frac{dF}{d\omega}$	Similarly $t^n \leftrightarrow \mathrm{j}^n\,\frac{d^nF}{d\omega^n}$
Conjugation: $f^*(t)$	$F^*(-\omega)$	If f real: $F(-\omega) = F^*(\omega)$
Even/odd: $f_e \leftrightarrow \Re\{F\}$, $f_o \leftrightarrow \mathrm{j}\,\Im\{F\}$		Symmetry links
Time reversal: $f(-t)$	$F(-\omega)$	Mirror in ω
Duality: $F(t)$	$2\pi\,f(-\omega)$	From this convention
Parseval/Plancherel	$\int \|f(t)\|^2 dt = \frac{1}{2\pi}\int \|F(\omega)\|^2 d\omega$	Energy conservation

Definitions (for this sheet).

- $\operatorname{sinc}(x) = \dfrac{\sin x}{x}$ with $\operatorname{sinc}(0) = 1$.
- $\operatorname{rect}(x) = 1$ for $|x| < \frac{1}{2}$ and 0 otherwise; $\operatorname{tri}(x) = \operatorname{rect} * \operatorname{rect}$.
- $\epsilon(t)$ is the unit step; $\operatorname{sgn}(t) = 2\epsilon(t) - 1$.
- p. v. denotes the Cauchy principal value.