Continuous (Unilateral) Laplace Transform Correspondence Table

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)\,\mathrm{e}^{-st}\,dt, \quad s \in \mathbb{C}$$

Notes: Unless stated otherwise, time-domain signals are assumed causal and multiplied by the step function $\epsilon(t)$. "ROC" denotes the region of convergence (typical/standard conditions).

A. Common Transform Pairs

Time domain $f(t)$	**Laplace** $F(s)$	**ROC (typ.)**	**Notes**
$\delta(t)$	1	all s	Dirac delta
$\delta(t-a),\ a \geq 0$	e^{-as}	all s	Time shift of impulse
$\epsilon(t)$	$\frac{1}{s}$	$\Re\{s\} > 0$	Step function
$t^n\,\epsilon(t),\ n \in \mathbb{N}_0$	$\frac{n!}{s^{n+1}}$	$\Re\{s\} > 0$	$n = 0, 1, 2, \ldots$
$\mathrm{e}^{at}\,\epsilon(t)$	$\frac{1}{s-a}$	$\Re\{s\} > \Re\{a\}$	Real/complex a
$\mathrm{e}^{at}t^n\,\epsilon(t)$	$\frac{n!}{(s-a)^{n+1}}$	$\Re\{s\} > \Re\{a\}$	Polynomial $\times$ exponential
$\sin(bt)\,\epsilon(t)$	$\frac{b}{s^2+b^2}$	$\Re\{s\} > 0$	$b \in \mathbb{R}$
$\cos(bt)\,\epsilon(t)$	$\frac{s}{s^2+b^2}$	$\Re\{s\} > 0$	$b \in \mathbb{R}$
$\mathrm{e}^{at}\sin(bt)\,\epsilon(t)$	$\frac{b}{(s-a)^2+b^2}$	$\Re\{s\} > \Re\{a\}$	Damped sinusoid
$\mathrm{e}^{at}\cos(bt)\,\epsilon(t)$	$\frac{s-a}{(s-a)^2+b^2}$	$\Re\{s\} > \Re\{a\}$	Damped cosine
$\sinh(bt)\,\epsilon(t)$	$\frac{b}{s^2-b^2}$	$\Re\{s\} > \lvert b\rvert$	$b \in \mathbb{R}$
$\cosh(bt)\,\epsilon(t)$	$\frac{s}{s^2-b^2}$	$\Re\{s\} > \lvert b\rvert$	$b \in \mathbb{R}$
$\frac{1}{k}\big(1-\mathrm{e}^{-kt}\big)\epsilon(t)$	$\frac{1}{s(s+k)}$	$\Re\{s\} > -k,\ k > 0$	1st-order step response
$(\epsilon(t)-\epsilon(t-T))$	$\frac{1-\mathrm{e}^{-sT}}{s}$	$\Re\{s\} > 0$	Rectangular pulse, width $T > 0$
$(t-T)\,\epsilon(t-T)$	$\frac{\mathrm{e}^{-sT}}{s^2}$	$\Re\{s\} > 0$	Delayed ramp
$\int_0^t f(\tau)\,d\tau$	$\frac{1}{s}F(s)$	depends on F	Time integral

Tip for students. Always check the ROC (e.g., $\Re\{s\} > 0$ for many causal signals) and remember that unilateral Laplace includes initial conditions in differentiation properties.

B. Useful Properties (Unilateral)

Operation in t	**Effect in s**	**Conditions / Notes**
Linearity: $\mathcal{L}\{af + bg\}$	$aF + bG$	$a, b \in \mathbb{C}$
Time shift (delay): $f(t-a)\,\epsilon(t-a)$	$\mathrm{e}^{-as}F(s)$	$a \geq 0$
Frequency shift: $\mathrm{e}^{at}f(t)$	$F(s-a)$	shifts the s-axis
Time scaling: $f(bt)$, $b > 0$	$\frac{1}{b}F\left(\frac{s}{b}\right)$	$b > 0$
Differentiation in t: $f'(t)$	$sF(s) - f(0^+)$	unilateral form (IC term)
Second derivative: $f''(t)$	$s^2F(s) - sf(0^+) - f'(0^+)$	include ICs
Integration in t: $\int_0^t f(\tau)d\tau$	$\frac{1}{s}F(s)$	as in Table A
Multiplication by t: $tf(t)$	$-\frac{d}{ds}F(s)$	frequency differentiation
Convolution: $(f * g)(t)$	$F(s)\,G(s)$	$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$
Initial value theorem	$f(0^+) = \lim_{s\to\infty} sF(s)$	if limit exists
Final value theorem	$f(\infty) = \lim_{s\to 0} sF(s)$	system stable, poles of $sF(s)$ in $\Re\{s\} < 0$ except possibly at $s = 0$