Discrete Fourier Transform (DFT) Correspondence Table

For a length-N sequence $x[n]$, $n = 0, \dots, N-1$:

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}, \quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi}{N}kn}, \quad k = 0, \dots, N-1.$$

Let $W_N := e^{-j2\pi/N}$ and all indices be understood mod N.

Notes: This sheet uses the standard engineering DFT with a $1/N$ factor in the inverse only. For real-valued $x[n]$, $X[0]$ and (if N even) $X[N/2]$ are real, and $X[k] = X^*[(N-k) \bmod N]$ (conjugate symmetry).

A. Common DFT Pairs

Time sequence $x[n]$	**DFT** $X[k]$	**Notes**
$\delta[n]$	1	Unit sample at n=0
$\delta[n-n_0]$	$e^{-j\frac{2\pi}{N}kn_0}$	Pure phase ramp
1 (constant)	$X[0] = N,\ X[k \neq 0] = 0$	DC only
$e^{j\frac{2\pi}{N}m_0 n}$	$N\,\delta[(k-m_0) \bmod N]$	Single spectral line at k=m_0
$\cos\left(\frac{2\pi}{N}m_0 n\right)$	$\frac{N}{2}\left[\delta[k-m_0] + \delta[k-(N-m_0)]\right]$	Two real symmetric lines
$\sin\left(\frac{2\pi}{N}m_0 n\right)$	$\frac{N}{2j}\left[\delta[k-m_0] - \delta[k-(N-m_0)]\right]$	Odd imaginary pair
Rectangular window: $w_L[n] = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{else} \end{cases}$	$W_L[k] = e^{-j\frac{\pi}{N}k(L-1)} \dfrac{\sin\left(\frac{\pi k L}{N}\right)}{\sin\left(\frac{\pi k}{N}\right)}$	Dirichlet kernel; $W_L[0] = L$
Shifted rectangle: $w_L[n-n_0]$	$e^{-j\frac{2\pi}{N}kn_0}\, W_L[k]$	Time shift $\Rightarrow$ linear phase

B. Core Properties (indices modulo N)

Time domain	**DFT domain**	**Comments**
Linearity: $ax[n] + by[n]$	$aX[k] + bY[k]$	$a, b \in \mathbb{C}$
Circular time shift: $x[(n-n_0) \bmod N]$	$e^{-j\frac{2\pi}{N}kn_0} X[k]$	Linear phase factor
Modulation: $e^{j\frac{2\pi}{N}k_0 n} x[n]$	$X[(k-k_0) \bmod N]$	Frequency shift by k_0 bins
Circular convolution: $(x \circledast y)[n]$	$X[k]\,Y[k]$	$(x \circledast y)[n] = \sum_m x[m] y[(n-m) \bmod N]$
Pointwise product: $x[n]y[n]$	$\frac{1}{N}(X * Y)[k]$	Circular convolution in k
Time reversal: $x[(-n) \bmod N]$	$X[(-k) \bmod N]$	Spectrum reversal
Conjugation: $x^*[n]$	$X^*[(-k) \bmod N]$	Conjugate symmetry link
Parseval/Plancherel	$\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \frac{1}{N}\sum_{k=0}^{N-1} \lvert X[k]\rvert^2$	Energy conservation
Correlation: $r_{xy}[n] = \sum_m x[m]\, y^*[(m-n) \bmod N]$	$R_{xy}[k] = X[k]\, Y^*[k]$	Circular correlation
Zero padding to $M > N$	Samples DTFT more densely	Improves spectral interpolation (not resolution of content)

Implementation tips.

- **Leakage:** Non-integer-bin sinusoids spread due to the rectangular window (Dirichlet sidelobes). Use windows (Hann, Hamming, Blackman) to trade mainlobe width for lower sidelobes.
- **Picket-fence effect:** Spectral peaks may fall between bins; zero-pad to sample the DTFT more densely and improve peak estimates.

- **Real signals:** Store only $k = 0, \dots, \lfloor N/2 \rfloor$ (others follow by conjugate symmetry).